Telecommunication Network Systems
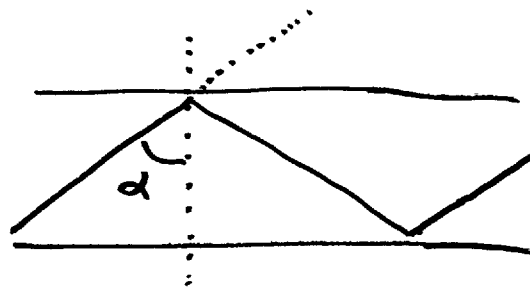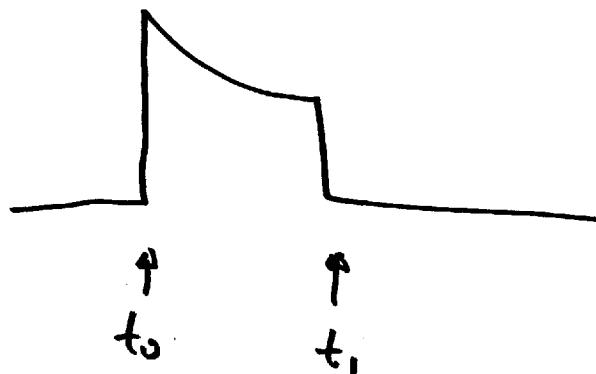
Modal Dispersion

1) Angle of total internal reflection

$$\alpha_c = \sin^{-1}\left(\frac{1}{n}\right) = 43.98^\circ$$

$$t_0 = \frac{L\,n}{c}, \qquad t_1 = \frac{L\,n}{c\,\sin(\alpha_c)}$$

For a 1 cm cable

$$t_0 = \frac{Ln}{c} = 48.0 \text{ ps}$$

$$t_1 = \frac{Ln}{c \sin(\alpha_c)} = \frac{Ln^2}{c} = 69.1 \text{ ps}$$

This means that they already start overlapping with next pulse. Overlapping effects occur for
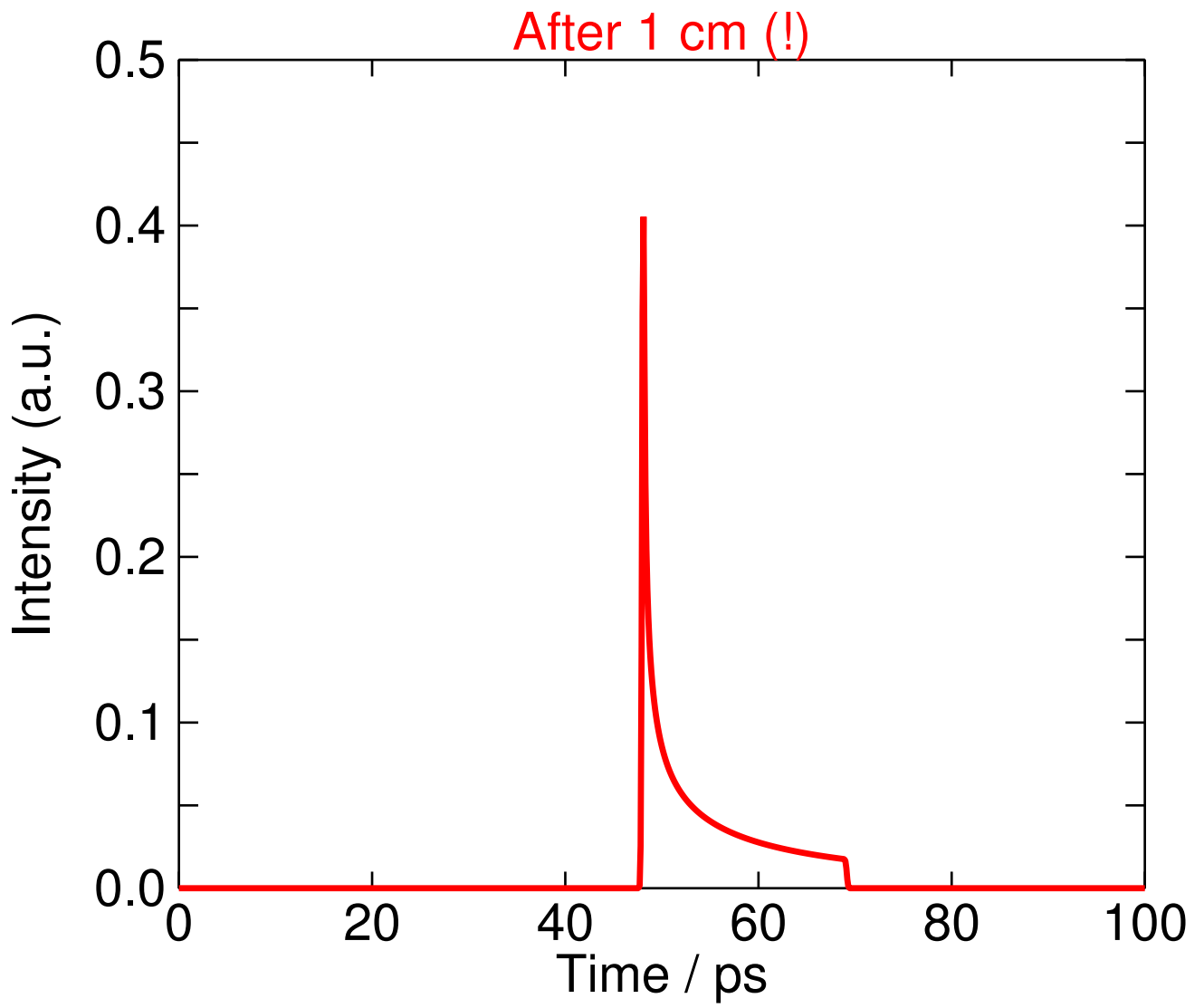
$$(t_1 - t_0) = 20 \text{ ps} - 7.5 \text{ ps} = 12.5 \text{ ps}$$

$$\frac{L}{c}(n^2 - n) = 12.5 \text{ ps}$$

$$L = \frac{12.5 \text{ ps} \cdot c}{n^2 - n} = 5.9 \text{ mm}$$

This is extremely small. Modal dispersion totally dominates everything!

See attachment for shape of wave of a single puls (δ function) ofkr 1 cm.

```pascal
PROGRAM Modedisp;

Uses XPSTool, PSMath;

Const Nsignal = 1000;
      Nangle = 1000;
      sigma = 0.1;

Var xlo, ylo, xhi, yhi: extended;
    x, y: extended;
    a, anglestep, alphac, L, c: extended;
    n: extended;
    t, tx, t0, t1: extended;
    i, j: integer;
    signal: array[0..Nsignal] of extended;
    f: text;

FUNCTION PSExp(r: extended): extended;
begin
  if r<-10000.0 then PSExp := 0
  else PSExp := Exp(r);
end;

begin
  PSFilename := 'MODEDISP.EPS';
  PSDisablePjotrSoft;
  EPSBoundingBox(1, 13, 18.8, 28);
  PSInit;
  xlo := 0; ylo := 0;
  xhi := 100; yhi := 0.5;
  PSDefineWorld(xlo, ylo, xhi, yhi);
  PSDefineWindow(4, 15, 18, 27);
  PSDrawBorder(6, 6, 6, -0.1);
  PSTextDirection(90);
  PSOutTextXY(xlo, (ylo+yhi)/2, '@cIntensity (a.u.)', 0, 2.5);
  PSTextDirection(0);
  PSOutTextXY((xlo+xhi)/2, ylo, '@cTime / ps', 0, -2);
  L := 1e-2;  { 1 cm }
  c := 3e8;
  n := 1.44;
  alphac := ArcSin(1/n);
  writeln('Alpha_c = ', alphac*180.0/Pi:0:3);
  t0 := L*n/c;
  t1 := L*n/c/Sin(alphac);
  writeln('t0 (ps) = ', t0/1e-12:0:2);
  writeln('t1 (ps) = ', t1/1e-12:0:2);
  for j := 0 to Nsignal do signal[j] := 0.0;
  anglestep := (Pi/2-alphac)/Nangle;
  for i := 0 to Nangle do
    begin
      a := Pi/2-i*anglestep;
      t := L*n/c/Sin(a)/1e-12;
      for j := 0 to Nsignal do
        begin
          tx := j*xhi/Nsignal;
          signal[j] := signal[j] + (1/(sigma*Sqrt(2*pi))) * PSExp( -Sqr(tx-t)/(2*Sqr(sigma)));
        end;
    end;
  PSSetColor(1, 0, 0);
  PSSetLineWidth(15);
  Assign(f, 'MODEDISP.OUT');
  Rewrite(f);
  for j := 0 to Nsignal do
    begin
      x := j*xhi/Nsignal;
      y := signal[j]/Nangle;
      PSGoto(x, y);
      Writeln(f, x, ' ', y);
      PSPenDown;
    end;
  PSPenUp;
  Close(f);
  PSOutTextXY((xhi+xlo)/2.0, yhi, '@cAfter 1 cm (!)', 0, 0.2);
  PSClose;
```